

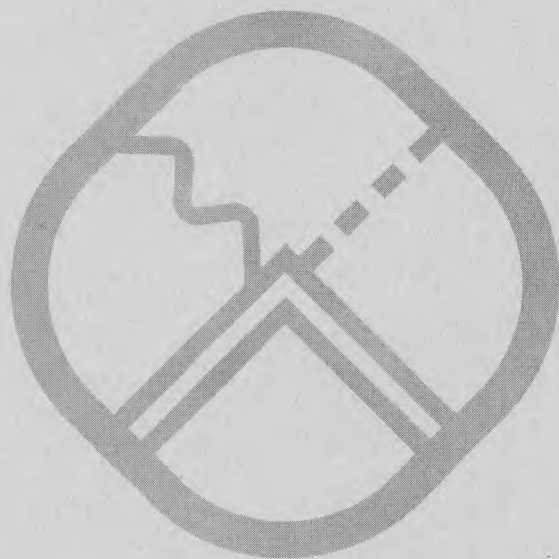
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K LEPTONIC DECAY AND PARTIALLY-CONSERVED CURRENTS

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Abstract

An operational definition for the partial conservation of the strangeness changing vector current is given and applied to leptonic K^+ and K_2^0 decay. The K^* resonance is explicitly included in the calculation and quantitative agreement with experiment is obtained. A detailed comparison with the K^+ data of Brown et. al. and Dobbs et. al. is given. Because of rapid variations of a form factor, it is found that the data of these two groups are not in contradiction. From the K_2^0 experiment of Luers et al., $I = 1/2$ and $3/2$ currents are seen to exist. $\Lambda\beta$ -decay is briefly considered. It is found that an explanation for the slowness of K leptonic decay and the vector part of $\Lambda\beta$ -decay may be connected with the partial conservation of the strangeness changing vector current.

I. DETERMINATION OF A THEORY FOR LEPTONIC K DECAY

One of the outstanding problems in the theory of weak interactions consists of finding a unifying principle for the strangeness changing and non-strangeness changing decays. Attempts to use a universal Fermi interaction or to generalize the idea of a conserved non-strangeness changing vector current have not been fruitful in the sense that an understanding of the experimental data has not been obtained.¹⁾ Furthermore, the ideas developed in attempting to explain the striking success of the Goldberger-Treiman formula in $\pi_{\mu\nu}$ decay²⁾ have not been carried over successfully into the theory of K decays.³⁾ Many of the present difficulties may well stem from our inability to give operational definitions to such concepts as a "partially-conserved current" and "universal interaction". In an attempt to sharpen our understanding of these terms, we have considered the leptonic decays of the K^+ .

The assumption is made that the $K^+ \rightarrow \ell^+ + \nu + \pi^0$ interaction is of the vector form, in which case we may write for the decay amplitude:

$$\langle \ell^+ \nu \pi^0 | K^+ \rangle = i \frac{G}{\sqrt{2}} \left(\frac{m_\nu}{E_\nu} \frac{m_\ell}{E_\ell} \right)^{1/2} \bar{\nu} \gamma_\alpha (1 + \gamma_5) \ell^+ \langle \pi^0 | s_\alpha^V | K^+ \rangle \quad (1)$$

where s_α^V is the strangeness changing vector current, and G is the weak interaction constant equal to 1.4×10^{-49} ergs \times cm³. By Lorentz invariance arguments, the matrix element $\langle \pi^0 | s_\alpha^V(0) | K^+ \rangle$ may be thrown into the form

$$\frac{1}{\sqrt{2}} \langle \pi^0 | s_\alpha^V(0) | K^+ \rangle = \frac{1}{2} (4 E_K E_\pi)^{-1/2} \left[(p_K + p_\pi)_\alpha f_+(s) + (p_K - p_\pi)_\alpha f_-(s) \right] \quad (2)$$

where $s = -(p_K - p_\pi)^2$. The four-momenta of the K and π are p_K and p_π . Using causality arguments, one can show that $f_+(s)$ and $(m_K^2 - m_\pi^2) f_+(s) + sf_-(s)$ satisfy subtracted dispersion relations.

It is not difficult to show that f_+ receives contributions (in the sense of dispersion theory) only from P-wave intermediate states. Also, since the matrix element $\langle \pi^0 | \partial_\alpha s_\alpha^V(0) | K^+ \rangle$ of the divergence of s_α^V is proportional to $(m_K^2 - m_\pi^2) f_+(s) + sf_-(s)$, it is precisely this combination of form factors that receives contributions from S-wave intermediate states. We now explicitly take into account the K^* ($K\pi$ spin 1^- resonance at 884 MeV), the only known particle or resonance that will contribute to our form factors.⁴⁾ Hence, we write

$$f_+(s) = \gamma \left\{ \frac{1}{1 - \frac{s}{M^2}} + v(s) \right\} \quad , \quad (3)$$

$$\langle \pi^0 | \partial_\alpha s_\alpha^V(0) | K^+ \rangle \propto \Delta m^2 f_+(s) + sf_-(s) = \gamma \Delta m^2 d(s) \quad (4)$$

where $\Delta m^2 = m_K^2 - m_\pi^2$, M is the mass of the K^* , and γ is a coupling constant that measures the strength of the $K^* K\pi$ interaction. Because we do not know of any zero mass particle that would give rise to poles in our form factors, we find

$$f_+(s) = \gamma \left\{ d(0) + \frac{s}{M^2} \frac{1}{1 - \frac{s}{M^2}} + v(s) - v(0) \right\} \quad (5)$$

$$\text{and } f_-(s) = -\gamma \frac{\Delta m^2}{M^2} \left\{ \frac{1}{1 - \frac{s}{M^2}} + \frac{M^2}{s} (v(s) - v(0)) - \frac{M^2}{s} (d(s) - d(0)) \right\}. \quad (6)$$

We now make the assumption that the current s_α^V is "partially conserved", by which we mean that $d(s)$ is slowly varying and $|d(s)| \ll 1$, in the physical region for s . This will justify neglecting the term $-\frac{M^2}{s} (d(s) - d(0))$ in the expression for $f_-(s)$. Note that this definition for the partial conservation of s_α^V differs from the ones usually adopted. Previously, the partial conservation of s_α^V has been taken to mean $\partial_\alpha s_\alpha^V = 0$ in the limit of some higher symmetry where baryon mass difference and meson mass difference vanish.⁵⁾ Alternative definitions have stipulated that $\langle \pi^0 | \partial_\alpha s_\alpha^V | K^+ \rangle \rightarrow 0$ as $s \rightarrow \infty$.⁶⁾ Since neither one of these latter two conditions is directly measurable in any decay experiment, we have chosen to redefine the concept of a partially conserved current.

In order to obtain an expression for f_+ and f_- that may be easily compared with experiment, we will make the rather crude approximation that $v(s) - v(0)$ is proportional to s . We may then write

$$f_+(s) = \lambda \left\{ \frac{s}{M^2} \frac{1}{1 - \frac{s}{M^2}} + \delta \right\} \quad (7)$$

$$\text{and } f_-(s) = -\lambda \frac{\Delta m^2}{M^2} \frac{1}{1 - \frac{s}{M^2}}, \quad (8)$$

$$\text{where } \delta = \frac{d(0)}{1 + M^2 \frac{dv(s)}{ds}} \bigg|_{s=0} \ll 1 \quad \text{and} \quad \lambda = \gamma \left(1 + M^2 \frac{dv(s)}{ds} \right) \bigg|_{s=0}.$$

We now have a two parameter theory. λ may be determined from the known K_{e3}^+ decay rate, while δ should follow from the observed $K_{\mu 3}^+/K_{e3}^+$ branching ratio.

II. PREDICTIONS AND EXPERIMENTAL CONFIRMATIONS OF THE THEORY

In Fig. 1 we have plotted the branching ratio $K_{\mu 3}/K_{e3}$ vs. δ . The curve is flat except for a very sharp rise near $\delta = 0$. The structure of this spike is a result of the combined hypotheses of a partially conserved current and "dominating" K^* pole. In the region of δ near the peak, not only the branching ratio, but also the spectra of all the particles, along with the longitudinal polarization of the μ , are extremely sensitive functions of δ . The size of δ should be compared with the pole term which has strength 1. A strictly conserved current would mean $\delta = 0$, a theoretically impossible situation ($\delta = 0$ also gives an incorrect branching ratio). Regardless of the value of δ , we may say in general that $K_{\mu 3}^+/K_{e3}^+ \leq 0.95$. The measured branching ratio is 0.96 ± 0.15 . This gives $\delta = -0.05^{+0.025}_{-0.05}$. Figure 2 shows some typical f_+ 's. Note that this form factor goes through zero in the physical region. Using the known rate for K_{e3}^+ decay, we may find λ^2 as a function of δ . The result is given in Fig. 3. Figure 4 shows the rate for K_{e3} as a function of δ , λ being held fixed.

If the theory is correct, it should be possible to fit both the π^0 and μ^+ energy spectra in $K_{\mu 3}^+$ decay by picking some value of δ in the range $-0.025 \leq \delta \leq -0.1$. Let us therefore look at Figs. 5 and 6 where the data from the experiment of Brown et al.⁷⁾ is displayed.

We see that the constant form factors ($\frac{f_-}{f_+} \equiv \xi = -9$) used by Dobbs et al. and Boyarski et al.⁸⁾ in their experiments cannot possibly fit either the observed π^0 or μ^+ energy distributions as measured by Brown et al. The curve corresponding to $\delta = -0.065$ gives reasonable agreement with experiment. Note that Brown et al. use two parameters in their fit while we use one. We find that δ comes out small compared to one, as our theory predicts.

Using the δ obtained from the experiment of Brown et al., we may compute what we would expect Dobbs et al. and Boyarski et al. to find in their experiments. The result is given in Fig. 7. Clearly, the form factors determined by Brown et al. do not fit the data of Dobbs et al. and Boyarski et al., while $\delta = -0.065$ gives a result consistent with experiment.

Because Dobbs et al. and Boyarski et al. measure only the upper part of the μ spectrum, while Brown et al. measure the π^0 energy spectrum and the bottom part of the μ spectrum, it is possible that the data of these three groups are not in contradiction. A contradiction will arise only if we assume that the form factors are essentially constant. Figures 8 and 9 give theoretical curves (without experimental biases) for the μ^+ and π^0 energy spectra.

Using the model with a fixed value for δ , the μ longitudinal polarization spectrum may be computed. Figure 10 gives some typical polarization curves. For large μ kinetic energies ($T_\mu > 110$ MeV), the polarization comes out negative for all reasonable δ (all values of δ compatible with the $K_{\mu 3}/K_{e 3}$ branching ratio). For intermediate values of T_μ ($35 \text{ MeV} < T_\mu < 75 \text{ MeV}$), the polarization is positive for all reasonable δ . For $T_\mu < 35 \text{ MeV}$, the polarization can be either positive or negative. An experiment measuring the polarization of high-energy μ 's has been proposed and carried out. The data are now being analyzed.

We would like to emphasize that certain very sensitive quantities, like the polarization of the μ in $K_{\mu 3}^+$ decay or the π^0 energy spectrum in $K_{e 3}^+$ decay, will not be very well determined within the framework of our approximations. Quadratic terms in s should also be included if we expect good agreement with experiment.

If we introduce a particle X to mediate the weak interactions, then we may summarize its effect by a change of form factors.

$$f_- \rightarrow f_- - \frac{\Delta m^2}{M_X^2} \frac{f_+}{1 - \frac{s}{M_X^2}} \quad f_+ \rightarrow \frac{1}{1 - \frac{s}{M_X^2}} f_+ ,$$

where M_X is the mass of the X . Because we lack detailed knowledge of $v(s)$ and $d(s)$, the leptonic decays of the K meson seem to be a poor place for isolating the effects of the X . Figure 11 gives some indication of the size of X effects.

The concept of a universal Fermi interaction has never been very well defined. For example, to test for universality in $K_{\ell 3}^+$ decay, it has been customary to consider $G f_+(0) \equiv S_V$ as an effective coupling constant. Since it turns out that $S_V^2 \ll G^2$, a universal form for the interaction is not apparent. However, if $f_+(s)$ is rapidly varying with s , then this test for universality may not be fair. Perhaps we should evaluate $G f_+(s)$ at a different value of s when forming S_V and making our comparison with G . The slowness of the leptonic decay of the K^+ might then be explained on the basis of a partially-conserved current. The rate is slow because the matrix element is of the order of $\langle \pi^0 | \partial_\alpha s_\alpha^V | K^+ \rangle$ which is a small quantity because s_α^V is partially conserved.

In concluding this section on K^+ decay, we would like to re-emphasize that the existence of the K^* along with the conjectured validity of a partially conserved current imply a profound deviation from what would be expected on the basis of phase-space arguments or almost constant form factors. If both the experiments of Brown et al. and Dobbs et al. prove to be correct, the hypothesis of almost constant form factors will no longer be tenable, while the assumption of a partially-conserved current may finally attain some degree of experimental confirmation.

As a further application of our hypothesis of a "dominating" K^* and a partially-conserved current, we have computed the form factors for neutral K leptonic decay and have compared our results for $K_2^0 \rightarrow e^+ + \nu + \pi^-$ with the experiment of Luers et al.⁹⁾

If we denote the corresponding form factors for K_2^0 leptonic decay by $h_+(s)$ and $h_-(s)$, we end up with the familiar form

$$h_+(s) = \lambda_2 \left\{ \frac{1}{1 - \frac{s}{M^2}} - 1 + \delta_2 \right\} , \quad (9)$$

$$h_-(s) = - \lambda_2 \frac{\Delta m^2}{M^2} \frac{1}{1 - \frac{s}{M^2}} . \quad (10)$$

If there was only an $I = 1/2$ current, then the spectra in K^+ and K_2^0 leptonic decay would be identical. In K_{e3}^+ decay we found that the π^0 energy spectrum had a zero when the kinetic energy of the π^0 was about 85 MeV (see Fig. 11). Since such a zero is not observed by Luers et al. in K_{2e3}^0 decay, we must have both $I = 3/2$ and $I = 1/2$ currents. The present data do not allow a useful determination of λ_2 and δ_2 .

It is interesting to note that if there exists a spin 1 $K\pi$ resonance other than the $K^{*,10)}$ then irrespective of the isotopic spin of this new particle, the form factors $f_+(s)$, $f_-(s)$, $h_+(s)$, $h_-(s)$ will still have the same effective representations (7), (8), (9), (10) if we neglect quadratic terms in s . Only the physical interpretation of λ , δ , λ_2 , and δ_2 will change. Hence, within the approximations made, our theory is not sensitive to the possible existence of other spin 1 $K\pi$ resonances.

Let us now briefly turn to the leptonic decay of the Λ . There, the strong interaction matrix elements of interest are

$$\begin{aligned}
 \langle p | s_{\mu}^V(0) | \Lambda \rangle &= \left(\frac{m_p m_{\Lambda}}{E_p E_{\Lambda}} \right)^{1/2} \bar{u}_p \left[i \gamma_{\mu} F_1(s) + i \frac{1}{2} [\gamma_{\mu}, \gamma_{\nu} s_{\nu}] F_2(s) + s_{\mu} F_3(s) \right] u_{\Lambda} \\
 \langle p | s_{\mu}^A(0) | \Lambda \rangle &= \left(\frac{m_p m_{\Lambda}}{E_p E_{\Lambda}} \right)^{1/2} \bar{u}_p \left[i \gamma_{\mu} \gamma_5 G_1(s) + i \frac{1}{2} [\gamma_{\mu}, \gamma_{\nu} s_{\nu}] \gamma_5 G_2(s) \right. \\
 &\quad \left. + \gamma_5 s_{\mu} G_3(s) \right] u_{\Lambda} ,
 \end{aligned}$$

where $s_{\mu} = (p_{\Lambda} - p_p)_{\mu}$ and $s = -s_{\mu} s_{\mu}$.

We consider the structure of s_{μ}^V . Proceeding as before, we find that F_1 receives only p-wave contributions and that

$$\langle p | \partial_{\alpha} s_{\alpha}^V(0) | \Lambda \rangle \propto \Delta m F_1(s) + s F_3(s) = \omega \Delta m D(s)$$

receives only s-wave contributions. $\Delta m = m_{\Lambda} - m_p$. Because we do not know of any zero mass particle that would give rise to poles in our form factors, we find

$$\begin{aligned}
 F_1(s) &= \omega \left\{ D(0) + \frac{s}{M^2} \frac{1}{1 - \frac{s}{M^2}} + V(s) - V(0) \right\} , \\
 F_3(s) &= -\omega \frac{\Delta m^2}{M^2} \left\{ \frac{1}{1 - \frac{s}{M^2}} + \frac{M^2}{s} (V(s) - V(0)) - \frac{M^2}{s} (D(s) - D(0)) \right\} ,
 \end{aligned}$$

where $V(s)$ represents all p-wave contributions to $F_1(s)$ other than those of the K^* . M is the mass of the K^* , and as in the case of K leptonic decay, $|D(s)|$ is expected to be small compared to 1 in the physical region for s .

The point we wish to stress is that while $F_3(s)$ may be treated as being essentially constant, $F_1(s)$ is a rapidly varying function of s and may even pass through zero in the physical region. Up to this time, it has been customary to take all form factors constant¹¹⁾ and, because of the small momentum transfers involved, $(m_\Lambda - m_p)^2 \leq s \leq m_e^2$, the terms containing $F_2(s)$ and $F_3(s)$ have been neglected compared to the term containing $F_1(s)$. It is quite possible that this procedure is not justified.

Once again the concept of a universal Fermi interaction is ill defined because of the rapid variation of $F_1(s)$. As in leptonic K decay, an explanation for the slowness of the vector part of $\Lambda\beta$ -decay may be connected with the partial conservation of s_α^V . Because of the lack of experimental evidence and the wealth of unknown constants in the form factors, we are not able to say more about the problem at this time.

ACKNOWLEDGMENTS

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FIGURE CAPTIONS

- Fig. 1: The branching ratio $K_{\mu 3}^+/K_{e 3}^+$ is plotted as a function of the parameter δ . The experimental value for the branching ratio of 0.96 ± 0.15 is represented by the horizontal solid and dashed lines. This indicates that the range of δ is limited to $-0.1 \leq \delta \leq -0.025$. The sharp rise is due to the zero in the form factor $f_+(s)$ which suppresses the $K_{e 3}^+$ rate more than the $K_{\mu 3}^+$ rate because $K_{e 3}^+$ depends only on $f_+(s)$, while $K_{\mu 3}^+$ depends on both $f_+(s)$ and $f_-(s)$.
- Fig. 2: The form factor $f_+(s)$ is given for three values of the parameter δ within the range determined by the branching ratio $K_{\mu 3}^+/K_{e 3}^+$. The coupling constant λ has been divided out of $f_+(s)$.
- Fig. 3: The effective coupling constant squared, λ^2 , is plotted as a function of δ . The experimental rate of $4.0 \times 10^6 \text{ sec}^{-1}$ for $K_{e 3}^+$ decay has been used. The dashed vertical lines indicate the restriction placed on δ by the known $K_{\mu 3}^+/K_{e 3}^+$ branching ratio.
- Fig. 4: The K leptonic decay rates are given as a function of δ with λ set equal to 1.
- Fig. 5: The histogram gives the π energy spectrum in the $K_{\mu 3}^+$ decay as measured by Brown et al.. The kinetic energy of the π is T_π . The smooth theoretical curves have been corrected for experimental biases. Brown et al. use a two-parameter fit, while the theory proposed in this paper uses the one-parameter δ . The curve labeled $\xi = -9$ is the constant form factor theory implied by the experiments of Dobbs et al. and Boyarski et al.

Fig. 6: The histogram gives the μ^+ energy spectrum in the $K_{\mu 3}^+$ decay as measured by Brown et al. The kinetic energy of the μ is T_μ . The smooth theoretical curves have been corrected for experimental biases. Brown et al. use a two-parameter fit, while the theory proposed in this paper uses the one-parameter δ .

Fig. 7: The experimental μ^+ energy spectrum, as measured by Dobbs et al., is represented by the histogram. The histogram has been corrected for experimental biases.

Fig. 8: The π energy spectrum predicted by various theories is given. Note the sensitivity of the spectrum to values of δ . The dip in the spectrum for the curve $\delta = -0.065$ is due to the zero of $f_+(s)$ in the physical region of s .

Fig. 9: The μ^+ energy spectrum predicted by various theories is given.

Fig. 10: The longitudinal polarization of the μ in $K_{\mu 3}^+$ decay is plotted as a function of μ kinetic energy. Although the polarization fluctuates wildly with small changes in δ , large μ energies always yield negative polarizations.

Fig. 11: The size of effects due to a vector boson X mediating the weak interactions is given for the electron energy spectrum in the $K_{e 3}^+$ decay. Although strong interactions could give rise to similar variations in the electron spectrum, the zero in the spectrum is a definite peculiarity of our theory arising from the zero of $f_+(s)$ in the physical region of s . A direct measurement of this spectrum would be a crucial test for the hypothesis of a partially conserved current.

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FIGURE 1.

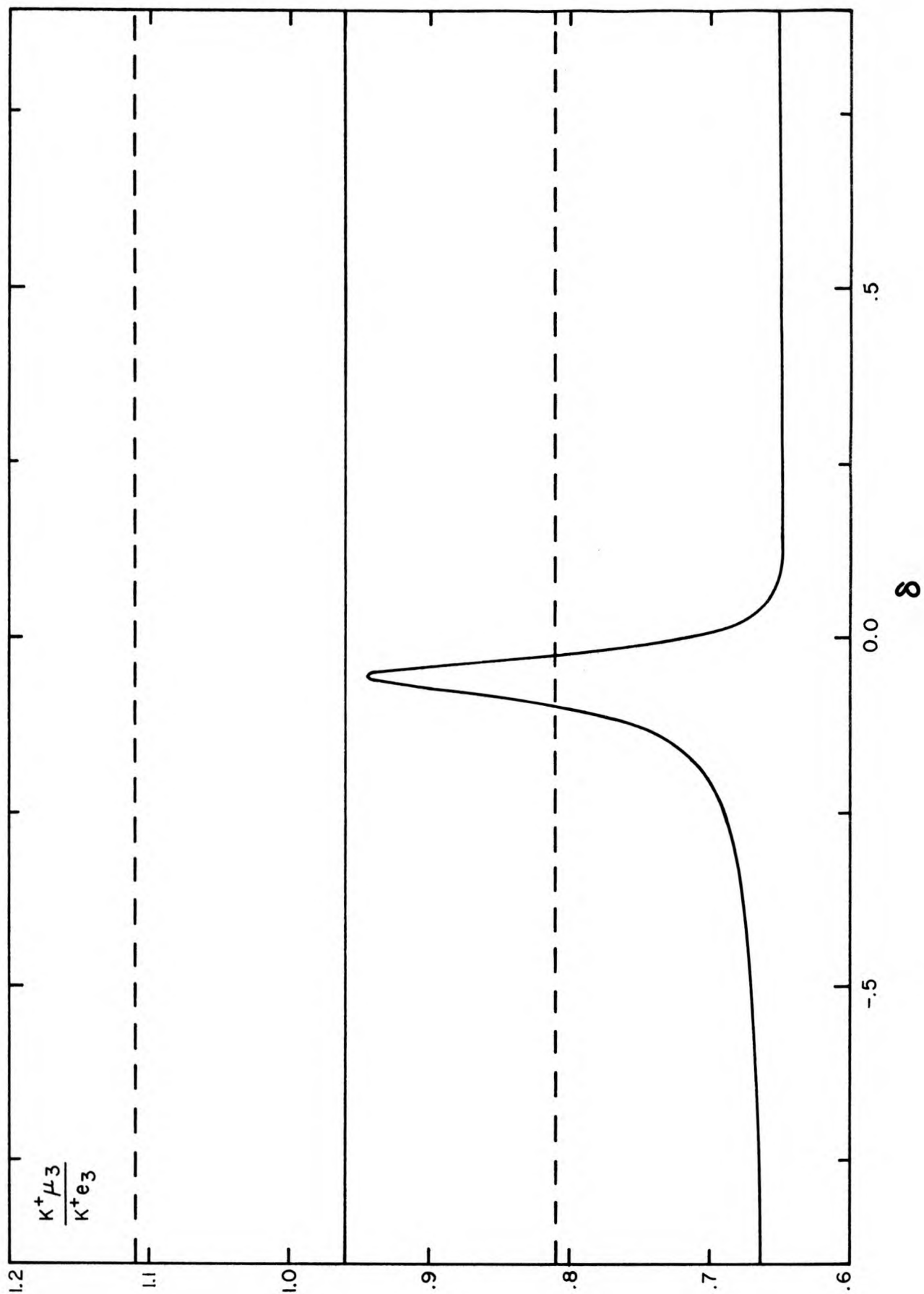


FIGURE 2

$\lambda' f_+(s)$

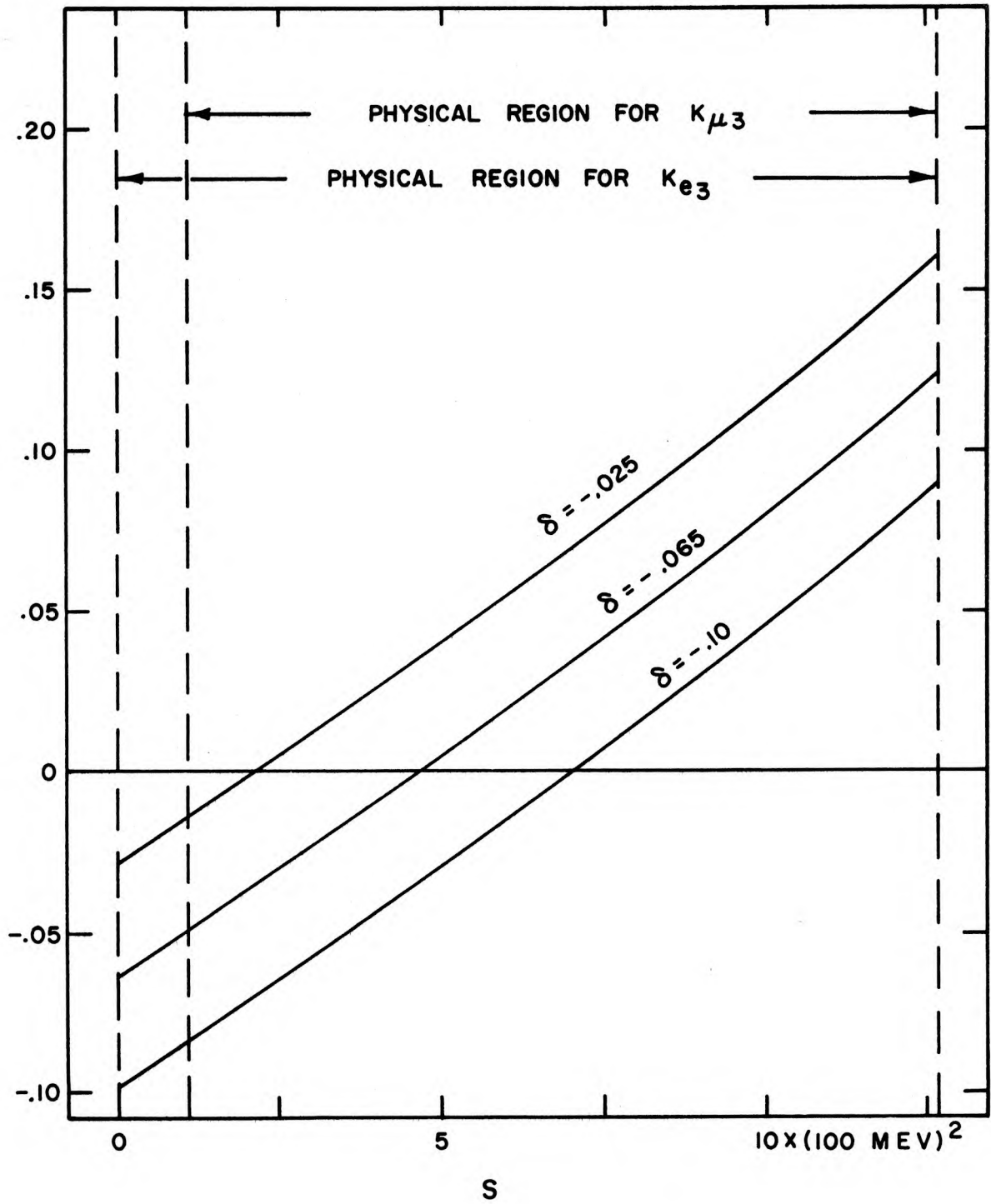


FIGURE 3

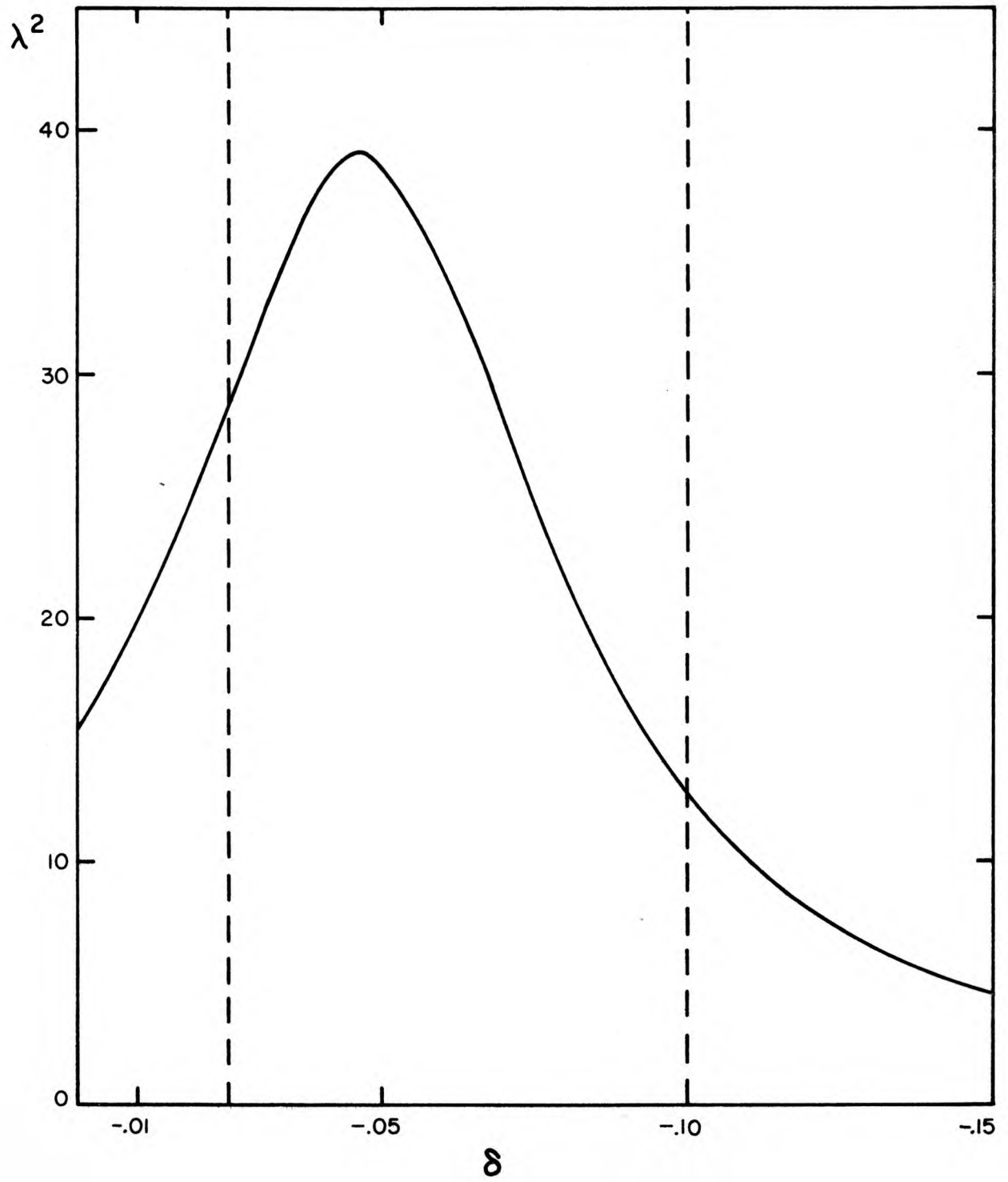
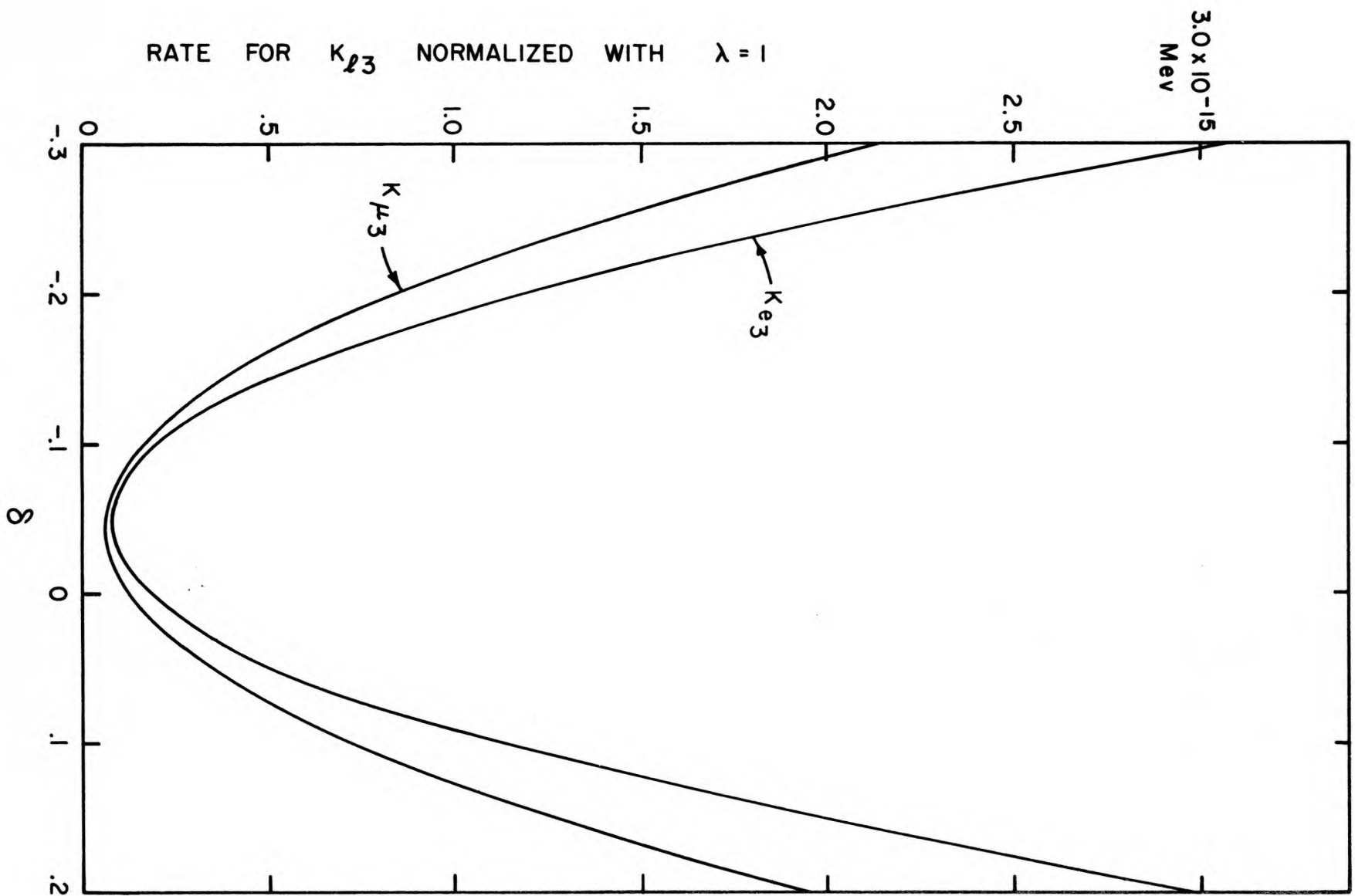


FIGURE 4



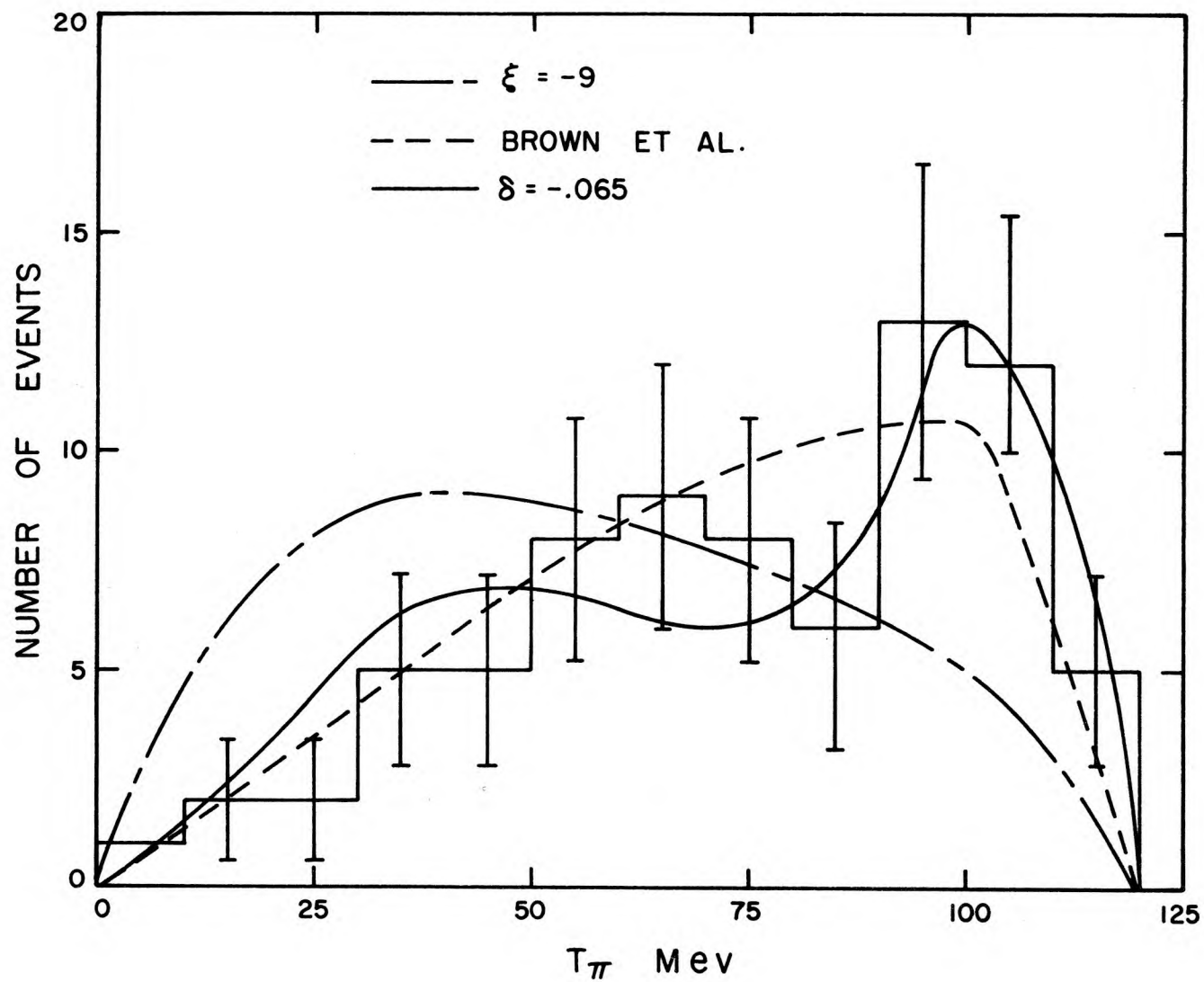


FIGURE 5

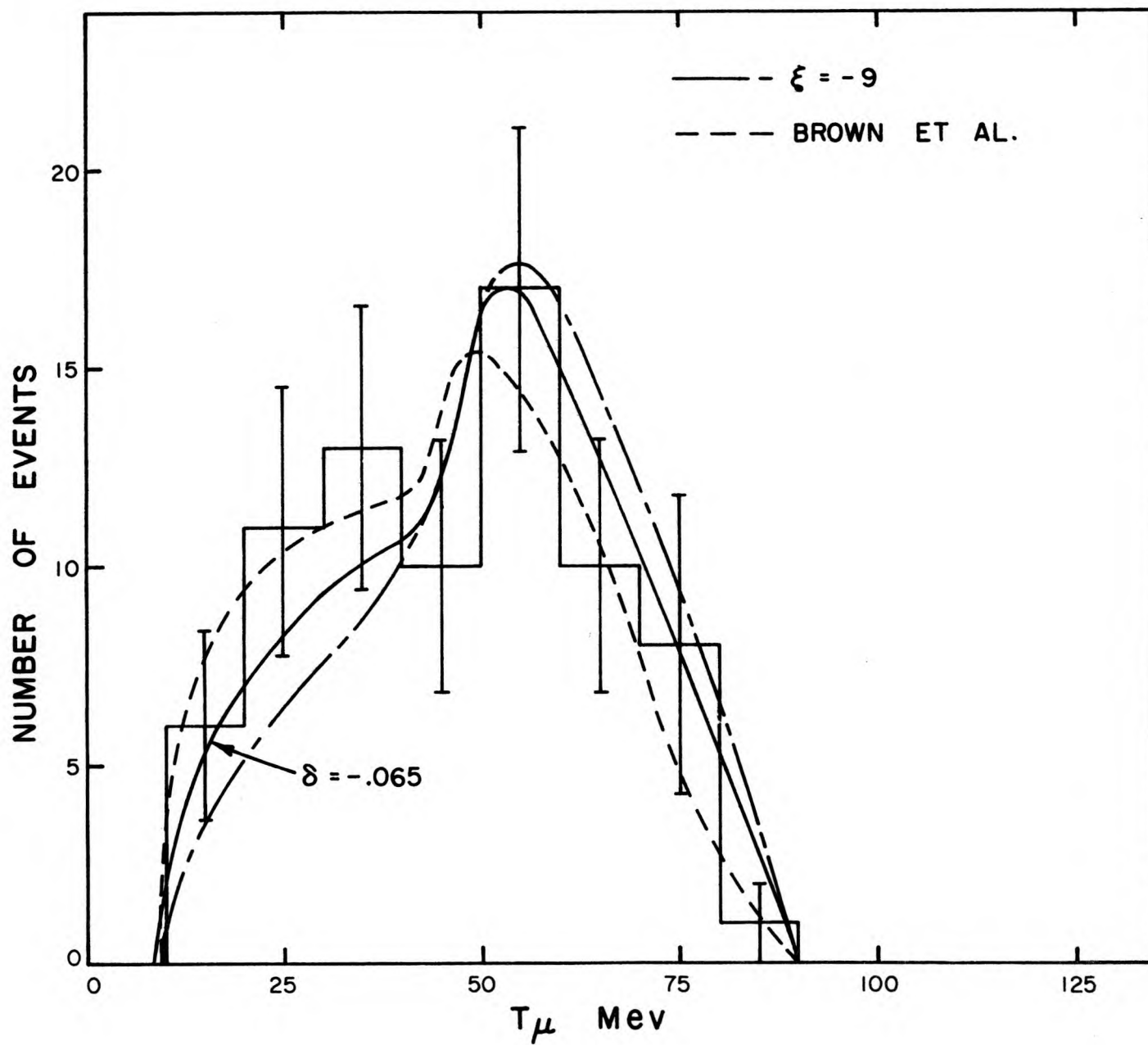
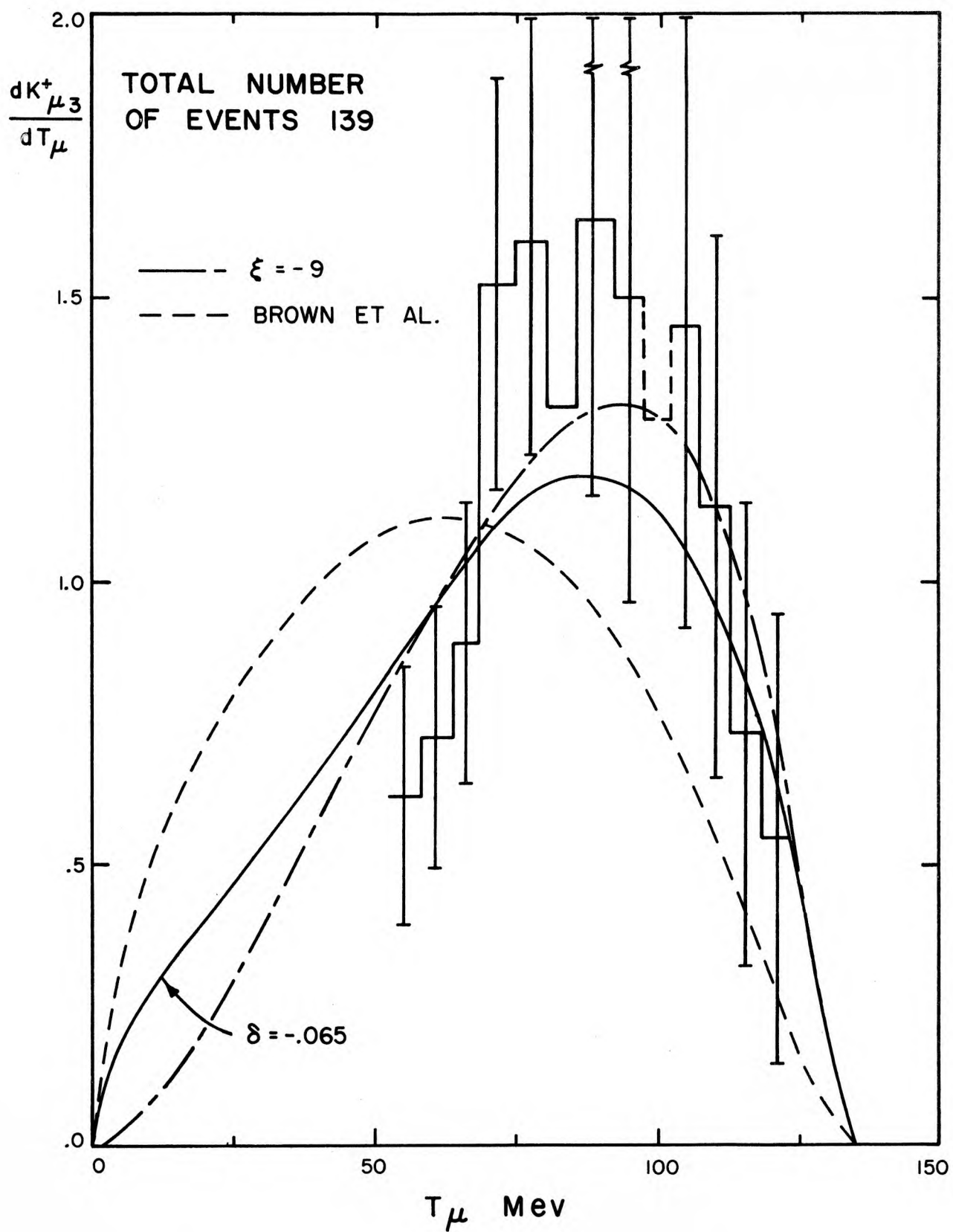


FIGURE 6

FIGURE 7



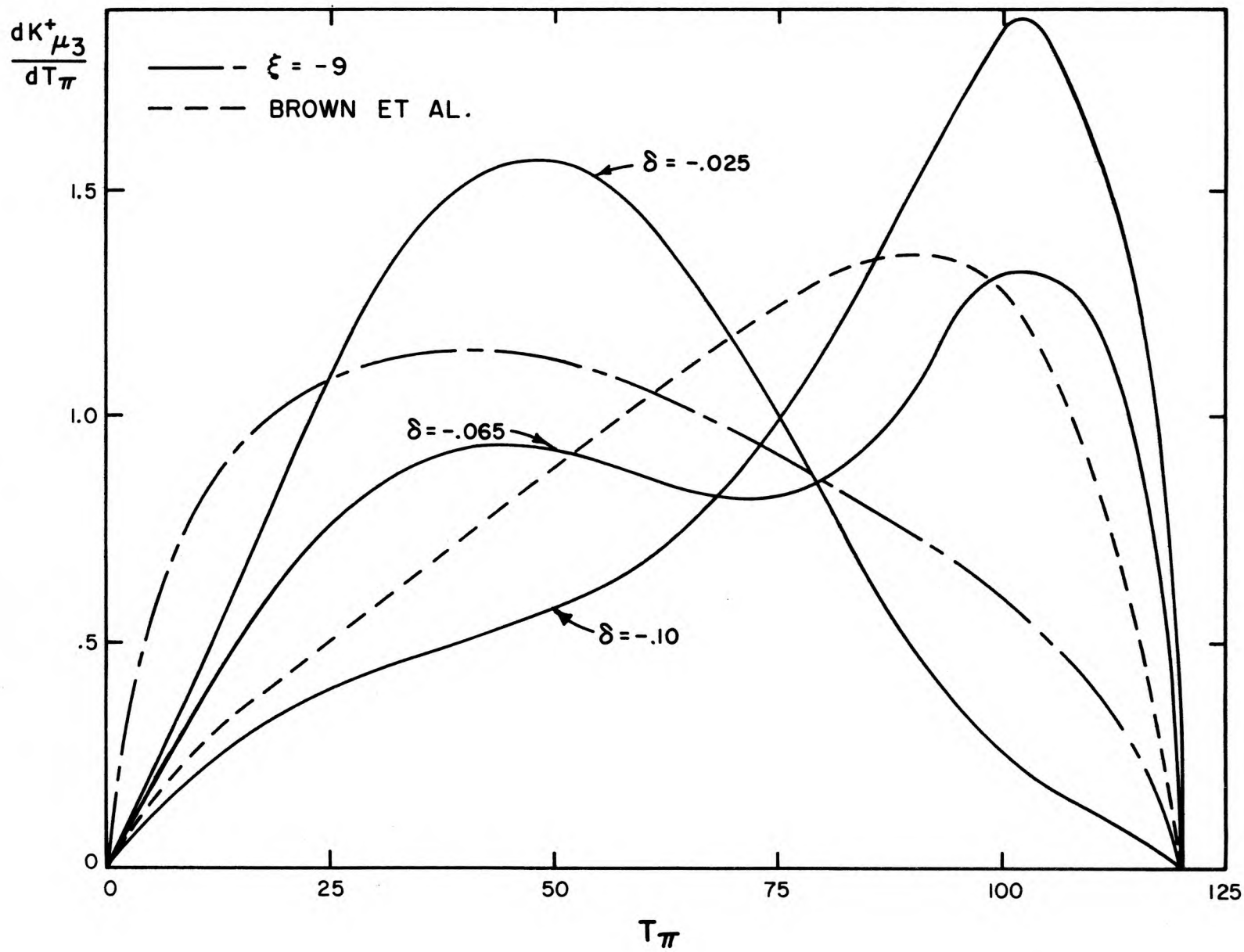


FIGURE 8

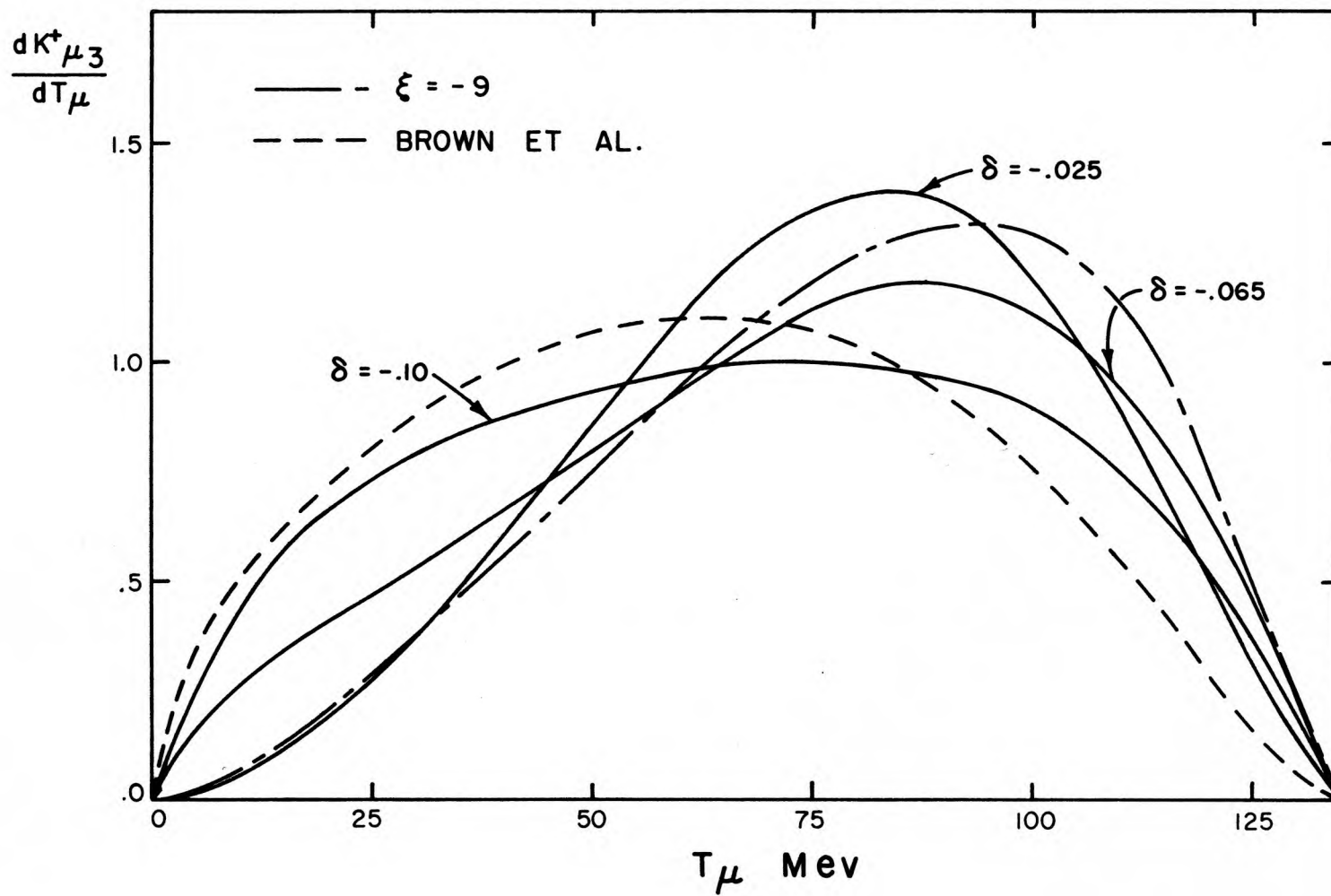


FIGURE 9

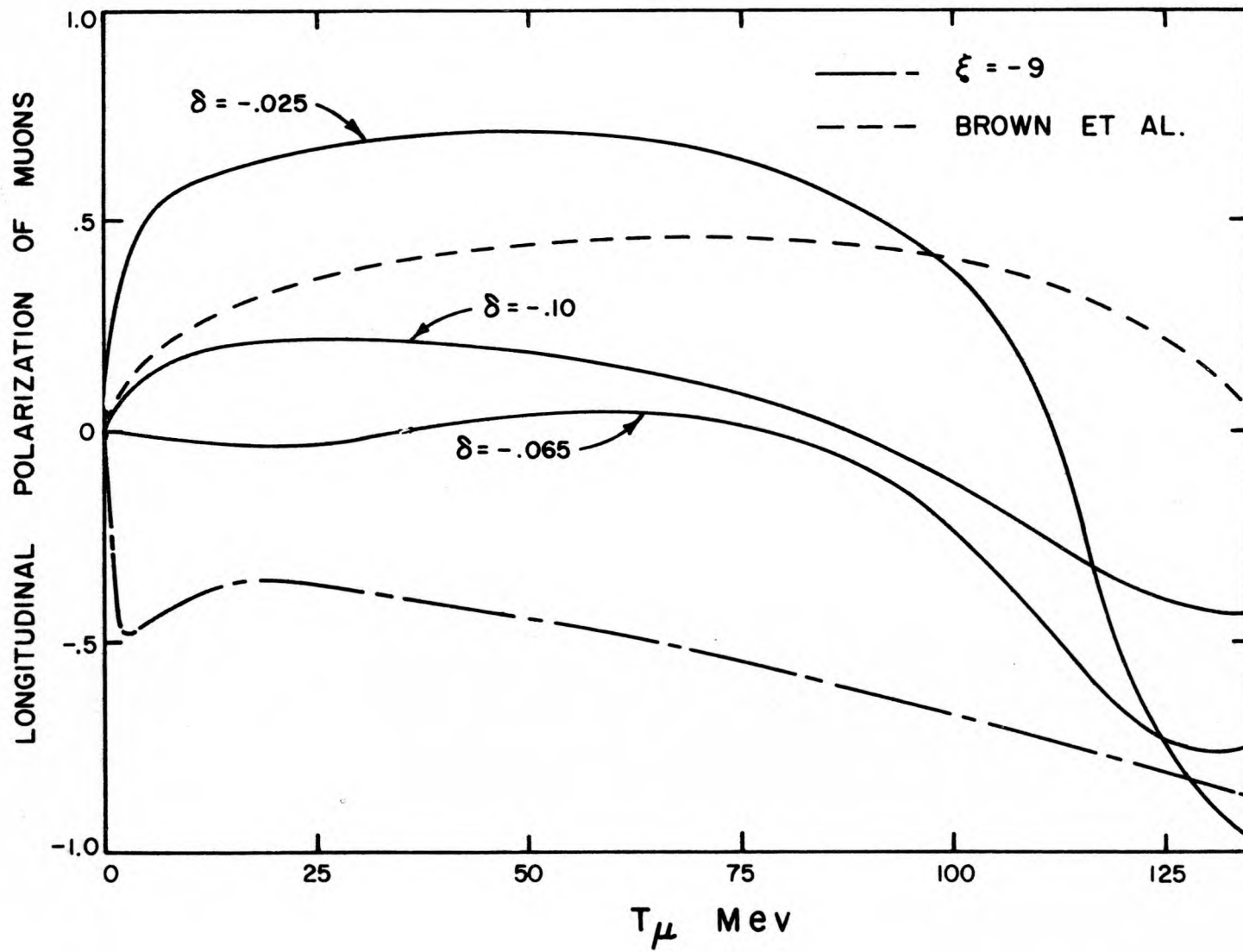


FIGURE 10

FIGURE 11

